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Bounding the Transient Response of Structures to Uncertain Disturbances

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An analytical method is presented for bounding the peak transient response of a structure to an arbitrary, dynamic input force vector of given total magnitude. Since the method is based on the linear operator norm for the multivariable impulse response of the structural system, it does not require either a simulation or a numerical search to bound the worst-case response. Instead, the bound is computed directly from the structural eigenmodes. The method reported in this paper is a generalization of existing bounding methods for single-input/single-output methods to the more practical case of vector-valued forces and vector-valued responses. In addition, the paper presents a method for computing one of the possible input force vector time histories that will excite a structural response equal to the bounded value. This worst-case input force is proposed as a candidate for force time histories to be used in structural qualification testing.

Introduction

DETERMINING the maximum response of a structure to all possible environmental excitations is a fundamental task in structural dynamics. Such an analysis is often a critical step in qualifying a spacecraft or satellite for flight by predicting the vibration integrity of aerospace vehicle components or electronic devices. If the transient stress response exceeds the stress safety margins or if the transient deformation interferes with the function of the structure, the structure must be redesigned.

An analysis of the transient response of a structure usually has three main steps. First, the environmental forces must be both cataloged and described in magnitude, time variation, spatial distribution, and direction. Second, the response of the structure to each anticipated force must be simulated, usually by integrating the modal equations of motion of a finite element model. Finally, the simulated responses must be examined to determine the peak time response of all locations within the structure to all of the anticipated environmental excitations.

Because many aerospace structures experience a variety of transient environmental inputs, analyzing all possible transient inputs and outputs may be time very consuming. This is further complicated by the fact that the exact environmental inputs are not always known at the time of the analysis. Instead, they often can only be characterized broadly, perhaps as having certain Fourier components or having a certain total root mean squared magnitude.

To address this problem, structural dynamicists have developed analytical methods that consider classes of input forces. A class of input forces is a set of force time histories that may be arbitrary in magnitude and duration within some bounding constraint. One constraint might be on the spectral magnitude of any force in the class; another might be the total time-domain power in the force signal. If the class of input forces is carefully defined, then it is possible to determine that force time history within a class that results in the worst-case response of the structure. This can be done without numerically simulating the response to each and every force history within the class. It can also be done without necessarily knowing the exact time history leading to the worst-case response. Instead, the bound can be determined using the definitions of the force class itself.

Previously derived methods of this type have considered variations in magnitude of the input and outputs of the structure in which the direction of the worst case input and/or output is precisely given. These single-input/single-output (SISO) methods include the method of shock spectra response,[1] the method of least favorable response,[2–5] and the method of matched filter theory.[6] All of these methods predict the worst-case time response of a single structural response variable to a force input applied to a single point. Each method also synthesizes an applied input that can excite the predicted worst-case response. It should be noted that these methods are entirely deterministic. That is, they do not consider the probability distribution of the input forces within the class definition. Instead, they attempt to compute the worst possible response to any force within the class.

Because these methods are SISO, the location and direction of the applied force and the set of structural outputs must be known. As such, it is not possible to allow the direction of the applied force or measured output to vary in defining the force class. This paper presents a method that overcomes this shortcoming by bounding the time response from vector-valued force inputs to vector-valued response outputs. The method is based on the theory of convolution operator norms for linear systems, as established in Ref. 7. The class of applied forces is the set of all force vectors with a given total integrated square magnitude. The method directly identifies the most sensitive response variable (displacement, acceleration, strain, etc.) within the structure. It is a single step computation and does not involve any numerical search for the worst-case input. If the vibration modes are known, the calculation can proceed directly from the eigenanalysis.

The paper is organized as follows. First, the previous methods of shock spectra response, least favorable response, and matched filter theory are reviewed. Then, the new worst-case analysis method is developed, and a method for computing the vector-valued force input time history that excites the worst-case response is also presented. An example application of the method is included at the end.

Existing SISO Bounding Methods

This section reviews previous methods for bounding the transient response of a structure to an unknown, scalar valued force input.

Shock Spectra and Least Favorable Response

Suppose that an input force $f(t)$ has a Fourier transform $F(\omega)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega x}\mathrm{d}t \tag{1}$$

One class of transient force inputs can be defined by the set of all $f(t)$ for which

$$|F(\omega)| \leq |G(\omega)| \tag{2}$$

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Such a class of environmental forces can be defined in this way by choosing $G(\omega)$ to be the envelop of the spectral magnitude of all expected environmental inputs.

The method of shock response spectra[1] and the method of least favorable response[2–5] use this particular class of force inputs. The method of shock response spectra synthesizes a force signal whose spectral content is the union of the peak magnitude spectral lines of all possible signals. This method has no rigorous mathematical basis; in particular, it is not necessarily a conservative analysis.

The method of least favorable response was developed to form a rigorous basis for the shock spectra approach. It bounds the maximum absolute value of a single response variable over all time from a bound in the magnitude of the applied force spectra. Suppose there exists a response of the structure $y(t)$. The method of least favorable response shows that the peak magnitude of $y(t)$ is bounded as follows:

$$\sup_{-\infty \leq t \leq \infty} |y(t)| \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|\,|H(\omega)|\,d\omega \tag{3}$$

in which $H(\omega)$ is the Fourier transform of the impulse response of the structure from the force f to the response y.

Matched Filter Theory

A similar result can be developed from matched filter theory. As presented in Ref. 6 for analyzing the peak gust response of aircraft, this method relies on a bound of the mean square magnitude of the impulse response of the structure from input to output. Rather than bounding the spectral magnitude of the input force, this method uses the response of an input prefilter to define the spectral content of all possible force inputs. In particular, the force $f(t)$ is assumed to be the response of a prefilter with Fourier transform $G(\omega)$ to an imaginary input $u(t)$ by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) U(\omega) e^{i\omega t}\,d\omega \tag{4}$$

in which $U(\omega)$ is the Fourier transform of $u(t)$. Additionally, $u(t)$ is arbitrary except that it must have unit total integrated magnitude,

$$\int_{-\infty}^{\infty} u^2(t)\,dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} U^*(\omega) U(\omega)\,d\omega = 1 \tag{5}$$

Notice that this constrains the square magnitude of the applied force $f(t)$ as well. With the prefilter $G(\omega)$ is known and for the class of inputs $u(t)$ defined by Eq. (5), the following must be true:

$$\int_{-\infty}^{\infty} f^2(t)\,dt \leq \frac{1}{2\pi} \int_{-\infty}^{\infty} G^*(\omega) G(\omega)\,d\omega \triangleq \sigma^2 \tag{6}$$

In other words, this method considers a class of applied forces that are constrained by a total integrated square magnitude:

$$\left[\int_{-\infty}^{\infty} f^2(t)\,dt\right]^{\frac{1}{2}} \leq \sigma \tag{7}$$

As shown in Ref. 6, the maximum response of the structure to any force satisfying the constraint (7) is given by

$$\max_{-\infty \leq t \leq \infty} |y(t)|^2 \leq \frac{\sigma^2}{2\pi} \int_{-\infty}^{\infty} H^*(\omega) H(\omega)\,d\omega \tag{8}$$

in which, again, $H(\omega)$ is the Fourier transform of the impulse response of the structure from the input force $f(t)$ to the response $y(t)$. Notice that this is the same as

$$\max_{-\infty \leq t \leq \infty} |y(t)| \leq \sigma \left[\int_{-\infty}^{\infty} h^2(t)\,dt\right]^{\frac{1}{2}} \tag{9}$$

in which $h(t)$ is the time domain impulse response or the inverse Fourier transform of $H(\omega)$. As will be shown, this result from matched filter theory is the scalar equivalent of the vector-valued case derived in this paper.

Proposed Vector-Valued Bounding Method

The difficulty in applying the matched filter theory result directly to a vector-valued case is that one has a choice of what to measure as the magnitude, or norm, of a vector. In particular, the norm of a vector can either be the maximum component value, or it can be the Euclidean norm of the vector. (In the scalar case, these are identical.) For this reason, it is necessary to use recent results in linear operator norm theory to analyze the worst-case vector-valued input and output.

Linear Operator Norms

Consider a linear, time-invariant system with a vector of applied inputs $\{f(t)\}$, a vector of response outputs $\{y(t)\}$, and a matrix of impulse responses $[h(t)]$. The element h_{ij} is the impulse response of the ith the element of $\{y(t)\}$ to the jth element of $\{f(t)\}$. The vector of responses is related to the vector of inputs by the convolution integral for the system,

$$\{y(t)\} = \int_{-\infty}^{\infty} [h(t-\tau)]\{f(\tau)\}\,d\tau \tag{10}$$

This convolution integral is a linear operator acting on $\{f(t)\}$ to produce $\{y(t)\}$.

Before proceeding, we need to define what is meant by the norm of a vector-valued function of time. Reference 7 provides the mathematical basis for this. For an n-dimensional real-valued vector function $\{f(t)\}$, we define the r norm to be

$$\|f(t)\|_r = \left[\sum_{i=1}^{n} |f_i(t)|^r\right]^{1/r} \tag{11}$$

Notice that this norm varies with time. Normally, only the 2 norm or the ∞ norm are of interest. When r is 2, this is the usual, Euclidean norm of the vector function:

$$\|f(t)\|_2 = \left[\sum_{i=1}^{n} |f_i(t)|^2\right]^{\frac{1}{2}} = \left[\{f(t)\}^T\{f(t)\}\right]^{\frac{1}{2}} \tag{12}$$

When r approaches ∞, the norm takes on the value of the maximum component of the vector $\{f(t)\}$ as a function of time:

$$\|f(t)\|_\infty = \max_{1 \leq i \leq n} |f_i(t)| \tag{13}$$

One can extend the definition of a vector norm to consider not only the norm across components as a function of time, but the norm across time as well.[7] We define the (p, r) norm of the vector-valued function $\{f(t)\}$ to be

$$\|f\|_{p,r} = \left[\int_{-\infty}^{\infty} [\|f(t)\|_r]^p\,dt\right]^{1/p} \tag{14}$$

Notice that this norm does not vary with time. When p approaches ∞, the (p, r) norm approaches the peak value of the r norm of the vector function $\{f(t)\}$ over all time:

$$\|f\|_{\infty,r} = \sup_{-\infty \leq t \leq \infty} \|f(t)\|_r \tag{15}$$

Usually, only the cases where p and r are 2 or ∞ are of interest. Consider first the (2, 2) norm. In that case, the norm is

$$\|f\|_{2,2} = \left[\int_{-\infty}^{\infty} (\|f(t)\|_2)^2\,dt\right]^{\frac{1}{2}} = \left[\int_{-\infty}^{\infty} \sum_{i=1}^{n} [f_i(t)]^2\,dt\right]^{\frac{1}{2}} \tag{16}$$

which is sometimes referred to as the root mean square magnitude of the vector function. If r is 2 and p approaches ∞, the (p, r) norm becomes the maximum value over all times of the Euclidean norm of the vector $\{f(t)\}$

$$\|f\|_{\infty,2} = \sup_{-\infty \le t \le \infty} \|f(t)\|_2 = \sup_{-\infty \le t \le \infty} \left[\sum_{i=1}^{n} [f_i(t)]^2\right]^{\frac{1}{2}} \tag{17}$$

If r approaches ∞ and p approaches ∞, this norm is the maximum value over all times of the maximum value of any component of the vector $\{f(t)\}$

$$\|f\|_{\infty,\infty} = \sup_{-\infty \le t \le \infty} \|f(t)\|_\infty = \sup_{-\infty \le t \le \infty} \max_{1 \le i \le n} |f_i(t)| \tag{18}$$

This mathematical framework allows us to analyze the worst-case response of a structure by looking at the norms of the convolution operator. Again, Ref. 7 provides the mathematical basis for this. We define the norm of the convolution operator

$$\int_{-\infty}^{\infty} h(t-\tau)()\,\mathrm{d}\tau \tag{19}$$

to be the maximum value of the (j, k) norm of $\{y(t)\}$ over all possible inputs $\{f(t)\}$ with (p, r) norm equal to 1. This is written as

$$\|h\|_{j,k}^{p,r} = \sup_{\|f\|_{p,r}=1} \|y\|_{j,k} \tag{20}$$

This norm is a scalar measure of the worst possible amplification of an input as it passes through the structural dynamic model. This norm expresses the amplification of an unknown force belonging to the set of forces $\{f(t)\}$ with (p, r) norm equal to 1.

In this paper, we restrict attention to all inputs with a given (2, 2) norm equal to σ. Ordinarily this can be expressed using a time-domain root means square value for the force, using Eq. (7). If the force spectral components are bounded, then σ must be computed using Eq. (6). In either case, we have the following theoretical result as an application of Theorem 1, Ref. 7.

Lemma: Define the matrix $[S]$

$$[S] = \int_{-\infty}^{\infty} [h(t)][h(t)]^T\,\mathrm{d}t \tag{21}$$

Result 1 (worst-case Euclidean norm): The peak transient Euclidean length $\|y\|_{\infty,2}$ of the vector of structural responses $\{y(t)\}$ to any force vector input $\{f(t)\}$ from the class of force histories that have a (2, 2) norm equal to σ will be bounded as follows:

$$\|y\|_{\infty,2} \le \sigma[\max \operatorname{eig}[S]]^{\frac{1}{2}} \tag{22}$$

Result 2 (worst case maximum component value): The maximum component absolute value $\|y\|_{\infty,\infty}$ of the vector of structural responses $\{y(t)\}$ to any force vector input $\{f(t)\}$ from the class of force histories that have a (2, 2) norm equal to σ will be bounded as follows:

$$\|y\|_{\infty,\infty} \le \sigma[\max \operatorname{diag}[S]]^{\frac{1}{2}} \tag{23}$$

Proof: The lemma is an application of Theorem 1, Ref. 7 to the case where σ is not 1. To do this, we need to establish that any force input in the set of vector inputs with (2, 2) norm equal to 1 has a corresponding force input in the set of vector inputs with (2, 2) norm equal to σ. To do this, consider any force input with (2, 2) norm equal to 1,

$$\|f\|_{2,2} = \left[\int_{-\infty}^{\infty} \sum_{i=1}^{n} [f_i(t)]^2\,\mathrm{d}t\right]^{\frac{1}{2}} = 1 \tag{24}$$

Form a new vector force input by multiplying $\{f(t)\}$ by σ. The (2, 2) norm of this new vector force input is σ:

$$\|\sigma f\|_{2,2} = \left[\int_{-\infty}^{\infty} \sum_{i=1}^{n} [\sigma f_i(t)]^2\,\mathrm{d}t\right]^{\frac{1}{2}} = \sigma\left[\int_{-\infty}^{\infty} \sum_{i=1}^{n} [f_i(t)]^2\,\mathrm{d}t\right]^{\frac{1}{2}} = \sigma \tag{25}$$

Similarly, if the response to $\{f(t)\}$ is $\{y(t)\}$, then the response to $\sigma\{f(t)\}$ is $\sigma\{y(t)\}$. The theorem is then established by scaling the results of Theorem 1, parts b and d from Ref. 7 by σ. Result 1 is σ times the bound of the convolution operator norm with $(p, r) = (2, 2)$ and $(j, k) = (\infty, 2)$. Result 2 is σ times the bound of the convolution operator norm with $(p, r) = (2, 2)$ and $(j, k) = (\infty, \infty)$. □

Notice that if the response is measured only by a single, scalar value, then results 1 and 2 are identical. In addition, if the input force is also a scalar value, then this becomes the same as the bound produced from matched filter theory.

To summarize, we have established worst-case bounds on the peak, time-varying, vector-valued response of a structure to an unknown vector-valued force input belonging to the class of inputs with a given (2, 2)-norm magnitude. This means that it is possible to bound the transient response of a structure without either knowing the actual force history or integrating the structural transient response to a given history. Whatever the actual peak transient response to any given force, it will be less than or equal to the bounds given. Furthermore, as shown in a subsequent section, there will always exist a particular force history with the given (2, 2) norm equal to σ for which the actual response to that force history exactly matches the worst-case bound for all other forces within that class.

Operator Norm for a Structure

The computation of the bounds requires the computation of the linear norm of the structural dynamic convolution operator. Consider a finite-order structural dynamic model with n second-order generalized coordinates and m inputs, $\{f(t)\}$. The equations of motion can be written as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = [D]\{f\} \tag{26}$$

Suppose that there is a vector of outputs $\{y(t)\}$ of length r that can be related to the generalized coordinates of the structure, $\{q(t)\}$, by

$$\{y\} = [C_q]\{q\} + [C_{\dot{q}}]\{\dot{q}\} \tag{27}$$

To calculate the matrix $[S]$, we first express these equations of motion in vector first-order (state-space) form. One formulation of this type is

$$\{\dot{x}\} = [A]\{x\} + [B]\{f\} \qquad \{y\} = [C]\{x\} \tag{28}$$

in which

$$\{x\} = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} \tag{29}$$

$$[A] = \begin{bmatrix} 0 & I \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \tag{30}$$

$$[B] = \begin{bmatrix} 0 \\ [D] \end{bmatrix} \tag{31}$$

$$[C] = [[C_q][C_{\dot{q}}]] \tag{32}$$

With this notation, Ref. 8 shows that the matrix $[S]$ is given by

$$[S] = [C][P][C]^T \tag{33}$$

in which $[P]$ is the controllability grammian of the system. It can be calculated by solving the linear Lyapunov equation

$$[A][P] + [P][A]^T + [B][B]^T = 0 \tag{34}$$

This direct method of calculating $[P]$ can be numerically ill conditioned for structural dynamic models with more than approximately 50 generalized coordinates. This situation is made worse by the presence of closely spaced modal frequencies, which is commonplace in complex, heterogenous structures. Fortunately, the solution to Eq. (34) is known in closed form in terms of the eigenmodes of the equations of motion (24). Suppose that the modal equations of motion of the structure can be obtained in the form

$$\{\ddot{\eta}\} + \mathrm{diag}[2\zeta_i\omega_i]\{\dot{\eta}\} + \mathrm{diag}\left[\omega_i^2\right]\{\eta\} = [\Phi]\{f\} \tag{35}$$

$$\{y\} = [\chi_d]\{\eta\} + [\chi_r]\{\dot{\eta}\} \tag{36}$$

in which $\{\eta(t)\}$ are the modal coordinates, and ω_i and ζ_i are the modal frequency and damping ratios. We define $\{b_i\}$ to be the ith row of the modal force matrix $[\Phi]$, where $\{c_{di}\}$ is the ith column of $[\chi_d]$, and $\{c_{ri}\}$ is the ith column of $[\chi_r]$. With these definitions, the controllability grammian is composed of $n \times n$, 2×2 blocks given by[8]

$$[P]_{ij} = \begin{bmatrix} 2\omega_i\omega_j(\zeta_j\omega_i + \zeta_i\omega_j) & \omega_j\left(\omega_j^2 - \omega_i^2\right) \\ -\omega_i\left(\omega_j^2 - \omega_i^2\right) & 2\omega_i\omega_j(\zeta_i\omega_i + \zeta_j\omega_j) \end{bmatrix} [\beta]\frac{1}{d_{ij}} \tag{37}$$

where

$$[\beta] = \{b_i\}\{b_j\}^T \tag{38}$$

$$d_{ij} = 4\omega_i\omega_j(\zeta_i\omega_i + \zeta_j\omega_j)(\zeta_j\omega_i + \zeta_i\omega_j) + \left(\omega_j^2 - \omega_i^2\right)^2 \tag{39}$$

Finally, the matrix $[S]$ is given by Eq. (33) with the following definition of $[C]$:

$$[C] = [[c_1]\cdots[c_n]] \tag{40}$$

with

$$[c_i] = \left[\{c_{ri}\}\left\{\frac{c_{di}}{\omega_i}\right\}\right] \tag{41}$$

Worst Possible Transient Force Input

As an analytical method, the preceding bound on the structural dynamic response provides a means for analyzing the worst-case transient response of a structure from a finite element model. Such an analysis would reveal the most critical stress components using only the modal information from the model. Also, it only requires that the class of force inputs can be characterized in terms of their given (2, 2) norm. A direct simulation of the model's response to a series of input forces within the class is not necessary to compute the worst-case transient bound.

It is often necessary, however, to qualify a structure by experimentally applying a force input to the structure that is a worst-case simulation of the operational environment. In this case, it might be necessary to know at least one force input time history that excites the structure so that the response is exactly equal to the bound. This force history could then be applied knowing that all other force histories with the same (2, 2) norm would excite a response less than or equal to the observed response, so long as the nonlinearities in the structure are weak. One such force history that causes the structure to meet its bounded response is derived next.

The more precise general derivation of Ref. 9 allows us to find a force input $\{f(t)\}$ that makes the magnitude of the response vector $\{y(t)\}$ reach the maximum value given by Eq. (20). The following discussion is a simplification and adaptation of this method to the problem considered here. After substituting the relation for $[S]$, Eq. (20) becomes

$$\|y\|_{\infty,2} = \sigma\left[\max \mathrm{eig}\int_{-\infty}^{\infty}[h(t)][h(t)]^T\,\mathrm{d}t\right]^{\frac{1}{2}} \tag{42}$$

Notice that the convolution integral for the linear system means

$$\{y(t)\} = \int_{-\infty}^{\infty}[h(t-\tau)]\{f(\tau)\}\,\mathrm{d}\tau \tag{43}$$

Equations (42) and (43) suggest that we look for force inputs of the form

$$\{f(t)\} = [h(-t)]^T\{a\} \tag{44}$$

in which $\{a\}$ is a vector of undetermined constants. The response at any time t is

$$\{y(t)\} = \int_{-\infty}^{\infty}[h(t-\tau)][h(-\tau)]^T\{a\}\,\mathrm{d}\tau \tag{45}$$

and the response at $t = 0$ is

$$\{y(0)\} = \int_{-\infty}^{\infty}[h(-\tau)][h(-\tau)]^T\{a\}\,\mathrm{d}\tau = \int_{-\infty}^{\infty}[h(t)][h(t)]^T\,\mathrm{d}t\{a\} = [S]\{a\} \tag{46}$$

The (2, 2) norm of the force input is

$$\|f\|_{2,2} = \left[\int_{-\infty}^{\infty}\{f(t)\}^T\{f(t)\}\,\mathrm{d}t\right]^{\frac{1}{2}} = \left[\int_{-\infty}^{\infty}\{a\}^T[h(-t)][h(-t)]^T\{a\}\,\mathrm{d}t\right]^{\frac{1}{2}} = [\{a\}^T[S]\{a\}]^{\frac{1}{2}} = \sigma \tag{47}$$

The matrix $[S]$ is positive semidefinite and symmetric, and so it has an orthonormal eigendecomposition given by

$$[S] = [E][\Lambda][E]^T \tag{48}$$

Equation (44) is maximized if we align the vector $\{a\}$ with the eigenvector of $[S]$ corresponding to the maximum eigenvalue $\Lambda_{\max}$,

$$\{a\} = a_0\{E_{\max}\} \tag{49}$$

in which a_0 is an undetermined constant. To satisfy Eq. (45), we choose

$$a_0 = \sigma/\sqrt{\Lambda_{\max}} \tag{50}$$

Finally, this particular worst-case force input is given by

$$\{f(t)\} = (\sigma/\sqrt{\Lambda_{\max}})[h(-t)]^T\{E_{\max}\} \tag{51}$$

To demonstrate this method, it will be applied to the two-input/single-output system diagrammed in Fig. 1. In this system, two rigid masses are connected by identical springs and dampers to each other and to a rigid wall. The input force vector has two components, each a lateral force acting on one of the masses. The output y is the relative displacement between the two masses. Each mass is 1 kg, each spring has a stiffness $(2\pi)^2$ N/m, and each dashpot has a value of 0.1 $k^{\frac{1}{2}}$.

Figures 2 and 3 plot the impulse response from each of the excitation forces to the output y. The worst-case response to a unit norm combination of the two inputs was calculated for this system using

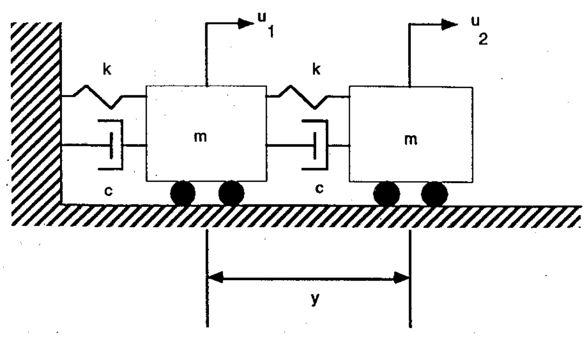

Fig. 1 Two-mass system studied in the example application.

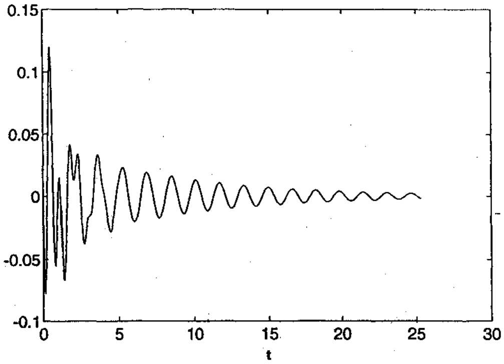

Fig. 2 Impulse response from u_1 to y.

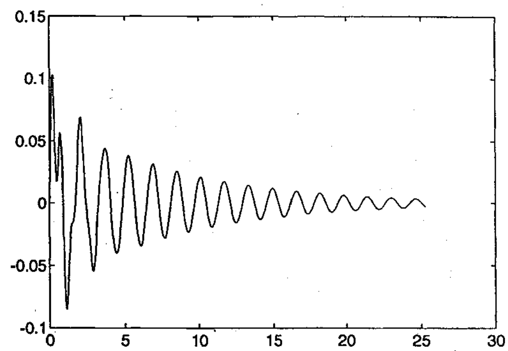

Fig. 3 Impulse response from u_2 to y.

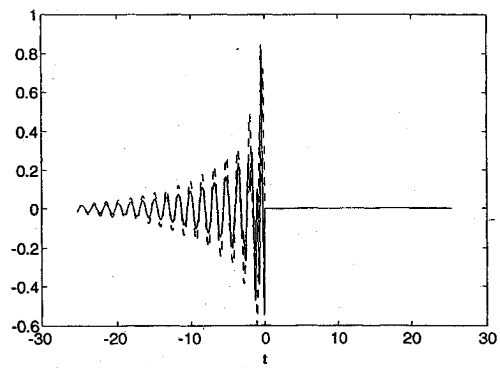

Fig. 4 Synthesized worst case input forces. The solid line is input 1 and the dotted line is input 2.

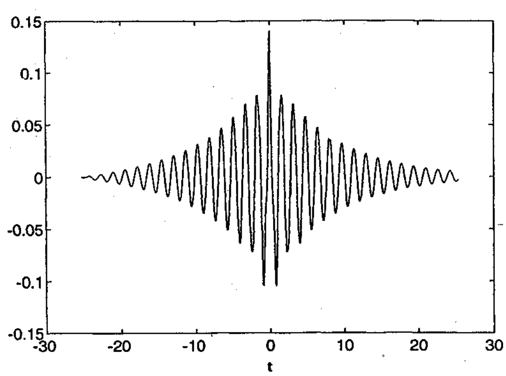

Fig. 5 Responses to the synthesized worst case input forces.

the Lyapunov formulation of Eqs. (33) and (34). This peak value was found to be 0.14. The worst-case combination of the two force inputs was numerically computed as described earlier. It is plotted in Fig. 4. Notice that the two force inputs resemble the backwards impulse response of the system, but they are not identical in phase. The worst-case input is a growing shock that is resonating the modes of the system.

The simulated response of the system to this worst-case input is shown in Fig. 5. The peak value of y occurs as expected at $t = 0$ and is the same as the predicted peak value from the operator norm analysis.

Conclusions

A method for predicting the maximum vector-valued response of a structure to an uncertain, vector-valued input has been developed. The method applies recent results from the theory of the norms of linear convolution operators. The class of force inputs considered are all of those with a given root mean square value, as determined either from a time power specification or a Fourier domain bound specification. For any transient force time history in this class, the peak magnitude or component value of the response of the structure will be less than or equal to given bounds. Two different response vector bounds were given, one when the magnitude of the response is chosen to be the Euclidean norm of the response vector and one for when the magnitude of the response is chosen to be the vector component with the maximum absolute value. These bounds can be used to analyze the worst possible transient response of a structural finite element model from modal information. It is not necessary to simulate the time response of the model to randomly selected forces to determine these bounds. For use in experimental qualification of a structure to a transient force, a vector-valued force input time history was presented that will make the structure attain the given bound. This worst-case transient disturbance varies from location to location within the structure and resembles the detransient or the impulse response for the dual of the structure from all inputs to the output of interest.

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